Tips $\lim_{t\to 0^-} e^{\frac{1-t}{t}}$ Concentrate

Example Find $\displaystyle\lim_{x\to 5} \arctan\left(\frac{(x+5)^2-100}{(2x-10)}\right)$

Solution:

$$\lim_{x\to 5}\frac{\overbrace{(x+5)^2-100}^{A^2\ -\ B^2}}{2x-10}$$

$\frac{0}{0}$ form : can't plug in.

Alternate:
$x^2+10x+25-100$
$x^2+10x-75$
$(x-5)(x+15)$

$$= \lim_{x\to 5}\frac{\overbrace{(x+5-10)(x+5+10)}^{(A-B)(A+B)}}{2(x-5)}$$

$$= \lim_{x\to 5}\frac{\cancel{(x-5)}(x+15)}{2\cancel{(x-5)}} = \lim_{x\to 5}\frac{x+15}{2}$$

$$= \frac{20}{2} = 10.$$

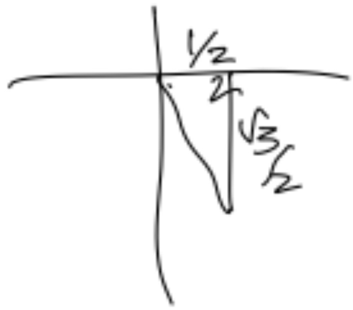
$$\therefore \lim_{x\to 5}\arctan\left(\frac{(x+5)^2-100}{2x-10}\right)$$

$$= \boxed{\arctan(10)} \approx 1.55$$

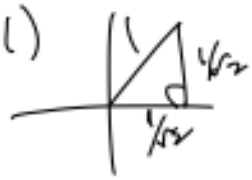
close to $\frac{\pi}{2}$

$\arctan(-\sqrt{3})$

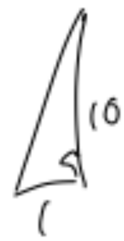
$= -\frac{\pi}{3}$



$\arctan(1)$
$= \frac{\pi}{4}$



$\arctan(10) = ?$



Draw $g(x)$ s.t. $g(2) = 1$ $\lim_{x \to -\infty} g(x) = -1$

$\lim_{x \to 2} g(x) = 3$

2 solutions to $g(x) = -4$

Example Draw an example of a function $y = F(x)$ such that

(3, −1) on graph

$F(3) = -1$,

As $x \to 2$, $y \to 4$

$\lim_{x \to 2} F(x) = 4$,

(2, −1) on graph

$F(2) = -1$,

$\lim_{x \to 0^+} F(x) = -\infty$,

$y \to -\infty$ as $x \to 0$ from + side

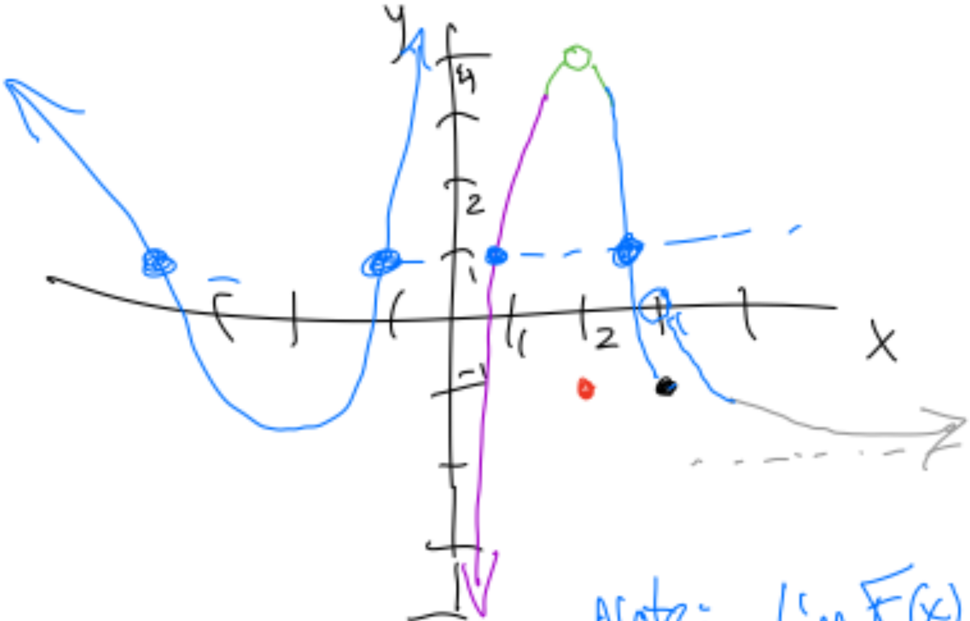
$\lim_{x \to \infty} F(x) = -2$

goes to $y = 2$ as x gets large

There are 4 solutions to $F(x) = 1$.

4 points where $y = 1$

''



Note: $\lim_{x\to 2} F(x) = 4$ and $F(2) = -1$ Tells us F is not continuous at $x=2$

$$\frac{(e^h-1)\sin(h)}{h^2} = \left(\frac{e^h-1}{h}\right)\left(\frac{\sin(h)}{h}\right)$$

$$\lim_{\theta\to 0} \frac{\theta^2\cot\theta}{\theta^2+3\theta} = \lim_{\theta\to 0} \frac{\theta^2\cos\theta}{(\theta^2+3\theta)(\sin\theta)}$$

$$\lim_{u\to 0} \frac{e^{u^2 \to 0} - 1}{\cos(3u) - 1}$$ mult by conjugate

$$\left(\frac{e^{u^2} - 1}{(u^2)}\right) \cdot (u^2) \frac{1}{\cos(3u) - 1}$$

$$\lim_{x\to 0} \frac{e^{x} - 1}{x} = 1$$

$$\frac{A}{B} = \frac{A}{1} \cdot \frac{1}{B}$$

Examples (More of them)

$$\lim_{u\to\infty} u^2 \sin^2\left(\frac{1}{3u + 2000}\right)$$

$$= \lim_{u\to\infty} u^2 \cdot \frac{\sin^2\left(\frac{1}{3u+2000}\right)}{\left(\frac{1}{3u+2000}\right)^2} \to 1^2 = 1$$

Form:

$u^2 \to +\infty$

$\frac{1}{3u+2000} \to 0^+$

$\sin\left(\frac{1}{3u+2000}\right) \to 0^+$

$\downarrow 0^+$

Thinking

$\infty \cdot 0$ form

undefined

$$= \lim_{u\to\infty} u^2 \cdot \frac{1^2}{(3u+2000)^2}$$

Special

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\Rightarrow \lim_{\theta\to 0} \frac{\sin^2\theta}{\theta^2} = 1^2 = 1$$

$$\left(\frac{\sin\theta}{\theta}\right)^2$$

$$= \lim_{u\to\infty} \frac{u^2}{(3u+2000)^2}$$

dominant

$$= \lim_{u\to\infty} \frac{\frac{1}{u^2}u^2}{\left(\frac{1}{u}\right)^2(3u+2000)^2}$$

$$= \lim_{u\to\infty} \frac{1}{\left(\left(\frac{1}{u}\right)(3u+2000)\right)^2} = \lim_{u\to\infty} \frac{1}{\left(3+\frac{2000}{u}\right)^2}$$

$\frac{2000}{u} \to 0$

$$= \boxed{\frac{1}{9}}$$

Bye!